EoR Experiment - Memo 4
Sensitivity and Calibration of PAPER

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Abstract

TBD

1 Random Noise Statistics

The electromagnetic field from the radio sky and the unwanted, but inevitable "noise" voltage generated in receiver elements have zero-mean, Gaussian statistics. The electromagnetic field arriving at our antenna *scriptE* is converted to voltage with (voltage) gain *scriptA* and impedance Z and added to the receiver noise. We describe these noisy voltage signals in a radio telescope receiver as "white," which means their Fourier transform is flat and contains no structure. We also describe it as "incoherent," which means one sample of a band-limited piece of the full spectrum has zero correlation with another sample separated by at least a time interval of $1/2B$ where B is the bandwidth. We explore the statistics of our sampled voltages with the monitor point of the CASPER correlator following the analog-to-digital converter (ADC). There is no Signal information in the individual voltage samples. Each independent sample is like the next: random, Gaussian variate. [Give reference such as ancient Bracewell article, or Davenport & Root.]

The Signal information we measure is obtained by looking at the variance of the voltage samples, i.e., power, which is v^2/R with R being 50 Ohms typically. The precision of a single estimate of power from a single sample is poor owing to the random nature and independence of samples. We have to integrate many samples to obtain higher precision. The statistics of the square of a Gaussian variate is exponential, which has rms/mean of 1. [This is an easy lesson in the conversion of probability density functions PDFs).] As we add samples together, the uncertainty in the mean as a ratio to the mean decreases as the square root of the number of adds. [This too is an even easier lesson in propagation of PDFs: the PDF of the sum of two variates is the convolution of their individual PDFs. Two e^{-x} PDFs convolved together leads to xe^{-x} statistics, which has rms/mean of $1/\sqrt(2)$.]

In summary when we integrate the power of receiver that receives a bandwidth B over time τ then the ratio of the rms of similar integrations, which is the uncertainty in an individual measurement, to the mean, which is what we are trying to measure, is $1/\sqrt{2B\tau}$. Why the factor of 2? The answer is that the electromagnetic fields we measure and the noise voltages generated in our receiver elements have sine and cosine components, two degrees of freedom. For great simplicity we often describes these signals as complex with amplitude and phase. This means the same thing. The complex representation comes with an understanding that for every $a_i \exp[j\omega t + \phi_i]$ complex phasor component to a spectrum of signals, where a_i, ϕ_i are the amplitude and phase at frequency ω, there is the "shadow" conjugate term at negative frequencies. That is, the voltages are indeed real! The signal spectrum is described as Hermitian, as will be so for the u, v sampling and its Fourier transform to sky intensity.

2 Radiometer Equation

The power P from a radio telescope receiver, or any electronic element, can be converted to a temperature, T, with the Rayleigh-Jeans relation.

$$P = kTB, \tag{1}$$

where k is Boltzmann's constant, $k = 1.38 \times 10^{-23}$ J K^{-1}. An example is the simplest electronic circuit: a resistor at temperature 300 K hooked up to to a detector in an impedance-matched circuit with a bandwidth of 1 MHz. This source resistor delivers 4 femtowatts of noise power to the load. This noise power comes from the thermodynamic jiggling of electrons in the resistor. Each moving electron delivers a "shot" of noise current (voltage) prior to its next collision.

A power detector on a radio antenna receiver will measure the sum of powers from the radiation received from sky, which we will characterize by antenna temperature T_a, and shot noise power from the receiver elements, which we will call T_r. We'll return to the explicit connection of T_a to the sky intensity field. Note that the "feed" element and attached receiver electronics must be impedance matched to the incident electromagnetic wave whose natural impedance is 377 Ohms; E/H has same units as V/I. In the following $T = T_\mathrm{a} + T_r$.

The connection between power received from the sky and that measured at the detector (ADC) and ultimately at the digital processing output is

$$P_\mathrm{a} = g_A^2 g_D^2 kTB, \tag{2}$$

where g_A is the analog voltage gain and g_D is the digital voltage gain. In subsequent discussion I will assume that g_D is eliminated by a priori normalization, and only use g for the gain. If we want to sample this power with an 8-bit ADC whose range is 0 to 1 Volt, then we'd like the rms voltage to be 4-bits, or so, and hence 16/64, or 250 mV (divide by 2 to split range to +/-). This 250 mV is converted to power via V^2/R relation with $R = 50$ Ohms to get 1.2 mW. A 100-MHz bandwidth signal then needs an analog power gain of approximately 3×10^9, or 95 dB.

Exercise: go through PAPER electronics and summarize electronic gain and losses of each element of the system–at 130 MHz and at 200 MHz. Conclude with looking at specifications of ADC to understand number of bits of expected noise quantization. Compare to real results using Aaron's ADC probing script.

3 Intensity, Flux Density, Power and Temperature Relations

3.1 Single Antenna

Consider first the power received by a single aperture antenna from the sky that is given by

$$P_{\nu,\mathrm{a}} = \int_{4\pi} d\Omega A(\theta,\phi) I_\nu(\theta,\phi), \tag{3}$$

where $A(\theta,\phi)$ is the "projected" area (m^2) of the antenna that collects incident radiation from direction (θ,ϕ), and $I_\nu(\theta,\phi)$ is the sky intensity in that direction. I has units of W m^{-2}Hz^{-1}sr^{-1}. $A(\theta,\phi)$ depends on a number of processes: diffraction and illumination and reflection. In subsequent discussion, we will drop the subscript ν, which indicates the "per Hertz" aspect of intensity and power. A uniform sky intensity, $I_\circ$ leads to a definition of "effective area (A_e)" and "effective beam size (Ω_e)," $P_\mathrm{a} = A_\mathrm{e}\Omega_\mathrm{e} I_\circ$. The quantity $A_\mathrm{e}\Omega_\mathrm{e}$ is known as "throughput" in optical/infrared jargon, and also "etendue." One tries to maximize the solid angle of the main beam of an antenna in relation to Ω_e, which includes sidelobes. In the Rayleigh-Jeans limit $I_\circ = 2kT_\circ/\lambda^2$, and then

$$P_\mathrm{a} = 2kT_\circ \tag{4}$$

with $\Omega_\mathrm{e} = \lambda^2/A_\mathrm{e}$. That is, the antenna temperature is the sky brightness temperature for a uniform sky just as if a resistor were heated to that incremental value at the antenna focus where conversion is done from electromagnetic waves to current in wires and amplifiers.

The antenna power gain, G, is a comparison of Ω_e to 4π.

$$G \equiv \frac{4\pi}{\Omega_\mathrm{e}} = \frac{4\pi A_e}{\lambda^2}. \tag{5}$$

When expressed in temperatures we can say

$$T_{\rm a} = \frac{A_e}{\lambda^2} \int_{4\pi} G_{\rm a}(\theta,\phi) T_{\rm b}(\theta,\phi), \tag{6}$$

where $G_{\rm a}(\theta,\phi) \equiv A(\theta,\phi)/A_e$ is the "relative power gain" of the antenna (also beam pattern and point spread function) and $T_{\rm b}$ is the "brightness temperature" of the sky; $G_{\rm a}(0,0) = 1.0$. A single source on the sky with uniform intensity $I_{\rm s}$ and small size $\Omega_{\rm s}$ has flux density S given by

$$S = I_{\rm s}\Omega_{\rm s}, \quad \text{and} \tag{7}$$

$$T_{\rm b,s} = \frac{S\lambda^2}{2k\Omega_{\rm s}}, \quad \text{and} \tag{8}$$

$$T_{\rm a,s} = (\frac{\Omega_{\rm s}}{\Omega_{\rm a}})T_{\rm b,s}. \tag{9}$$

The point of the final equation is that there is "beam dilution:" the antenna temperature, which sets the power delivered to the detector, from an unresolved source, which is one much smaller than the resolving power of the antenna, is reduced by the ratio of source solid angle to the beam solid angle.

For the dipoles used in PAPER, $A_{\rm e}$ is approximately 1.5 m^2. The receiver temperature is around 100 K and the sky brightness temperature varies over large regions seen by the dipoles from 100 K to 400 K around 150 MHz. [Make this more specific from Haslam extrapolation to 150 MHz, and Nicole's integration.] Astronomers often characterize the sensitivity of a radio antenna by its system equivalent flux density (SEFD). For our dipoles this is $T_{\rm sys}2k/A_{\rm e}$, or 400,000 Jy to 1,000,000 Jy using range quoted above; i.e., huge! We do detect strong sources like Cygnus owing to the "root $2B\tau$" factor.

Noise in power detectors, and correlators as we will discuss, is reduced from the SEFD, or $T_{\rm sys}$, value by the number of independent noise samples that are averaged. There are two degrees of freedom, amplitude and phase, for each independent Fourier component, whose number per integration is given by bt.

$$\delta S = \frac{\rm SEFD}{\sqrt{2B\tau}}, \quad \text{and} \tag{10}$$

$$\delta T = \frac{T_{\rm sys}}{\sqrt{2B\tau}}. \tag{11}$$

With more precision, the factor of "2" depends on actual detection scheme with such issues as low-pass filter characteristics of integration and potential switching techinques.

With the PAPER/8 correlator we look at a total bandwidth of 150 MHz at a center frequency of 150 MHz with 256 channels, which provides ~600-kHz bandwidth per channel, and integrate for 3.5 seconds; $2bt$ is then approximately 2000. The expected noise per sample is then 200 Jy to 500 Jy. Thus, Cygnus near 10,000 Jy stands out loud and clear, while the Crab at 1500 Jy is just visible in fringe phase without averaging. The Sun can flare into MJy range, and so can be seen as noticeble increment in total power; i.e., in autocorrelations.

Also, the noise in autocorrelations correspond to antenna temperature fluctuations of 20 mK to 50 mK. To get the fluctuations down to the mK level, which is characteristic of the the EoR Signal level, requires an additional integration of 400 to 2500 integrations, which is modest. The issue about achieving mK or lower noise level is one of stability. One only achieves a lower noise level from longer integrations for a zero mean, Gaussian process; i.e., one without RFI and gain, or other, instabilities.

3.2 Interferometer Array

The discussion above applies to the compound antenna–the interferometer array. We are particularly interested in brightness temperature sensitivity. With the interferometer array a beam is synthesized by phasing and summing all interference products, $N(N-1)/2$ for an N-element array. This produces a beam with $\Omega_{\mathrm{e}} << \lambda^2/A_{\mathrm{e}}$, where $A_{\mathrm{e}} = ND_{\mathrm{e}}^2$ and D_{e} is the effective diameter of the individual element. That is, the "throughput" is reduced for an underfilled aperture. The synthesized beam is of order $\lambda^2/b_{\mathrm{max}}^2$; the exact relation depends on the weighting applied in the imaging algorithm. Thus we find that

$$T_{\mathrm{a}} \simeq N(\frac{d_{\mathrm{e}}^2}{b_{\mathrm{max}}^2})T_{\mathrm{b}}, \tag{12}$$

while the noise in the image is that of the single array antenna element reduced by the number of interference products,

$$\delta T = \frac{T_{\mathrm{sys}}}{\sqrt{(N(N-1)B\tau}}. \tag{13}$$

Eq. 12 is to be compared with Eq. 4 to understand the reduction in brightness temperature sensitivity of an unfilled interferometer array in comparison to the high throughput single antenna.

With PAPER we are working toward an array of 100 elements with each having an A_{e} of 20 m^2. If we place these within a b_{max} of 600 m, then a 1 mK brightness temperature 1σ noise level (Eq. 13) in an average image in a 1-MHz bandwidth (b) channel requires integration time of xx seconds, or xx days at 3h per day. Note that integration time per day without phase-tracking is limited by $\lambda/d_{\mathrm{e,EW}}$, the East-West effective diameter of individual elements.

3.3 MIRIAD <obsrms>

The MIRIAD routine <obsrms> provides a ready means to explore brightness temperature sensitivities for interferometer arrays. The ratio of keyword inputs *tsys* and *jyperk* is the quantity *SEFD* discussed above. An alternative approach to *jyperk* is to provide the *antdiam*, which may be a single value (in m), or that and a second value for the efficiency (default 0.6). The radio frequency, *freq*, is given in GHz, or you can provide the wavelength using *lambda*, in mm. The antenna array and imaging specifics are not required; these issues are summarized in *theta*, the beamsize in arcsec, which is a single value for circular beam, or two values for elliptical beam. While the array distribution is not specified, the number of antennas is using *nants*; *nants, antdiam* (or jyperk), and *beamsize* effectively provide a measure of the sparseness of the array. Further parameters for this sensitivity calculation are *bw*, the bandwidth in MHz, and *inttime*, the integration time in minutes. [SI not spoken here in favor of nominally small-number values.] There are a couple other keywords that are not relevant to our application.

Example: *tsys=200 antdiam=1.4 freq=0.15 nants=8 inttime=120 theta=1800* tells us that in 2h with our current 8-dipole array we should be able to achieve 1 Jy noise level in imaging with 0.5d resolution. This corresponds to an rms brightness temperature of 18 K. With 6h per day and 6d, we could achieve 3 K. If we lowered resolution of array to achieve 1d resolution, then a similar week-long integration might break the 1 K boundary. Or we can open the bandwidth to 50 MHz and try for 1 K in 1d. A reasonable goal for the PAPER/WA data is imaging at the 1 K level—then we just have three orders of magnitude of sensitivity to go! We can't boost bandwidth beyond 1 MHz too far or we won't be sensitive to our goal of redshifted 21cm signals. We have plans ' of boosting antenna gain by 4 (effective diameter by 2), and we definitely want to buildout the number of antennas by ~ 16. The higher antenna gain means less integration per day, and so we must intergrate more days. A factor of 30 improvement then would get us half way to goal...on a log plot! The next step would be even higher gain by factor of 30 via more elements and larger correlator, or more elements phased together to look at smaller patch along equator or at pole.

Table 1: 3C Catalog (159 MHz) Sources Above 100 Jy

3C#	hh	mm	ss	dd	mm	Jy	Size ($'$x$'$)	Other Names Other Names
10	00	22	37.	+63	52.	110	5.4x0.5	Tycho Brahe's SNR
123	04	33	55.	+29	35.	204	<1	W7
144	05	31	31.5	+21	59.2	1500		Std Source: Crab Nebula (NGC 1952)
157	06	14	36.	+22	43.	270	>30	W14, IC443
163	06	29	18.	+05	12.	450	60	MSH 06+08, W16, Rosette Neb. (NGC 2237)
218	09	15	42.	-11	53.	210	2.3x0.5	MSH 09-14, DWD
274	12	28	18.	+12	40.1	1100		Std Source: NGC4486(M87)
348	16	48	43.	+05	10.	300	2.3x0.5	MSH 16+010 W20, DWD
353	17	17	58.	-00	52.	180	3.5x0.5	MSH 17-06, W21, DWD
392	18	53	35.	+01	15.	680	<45	MSH 18+011, W44
405	19	57	44.5	+40	35.	8600		Std Source: Cygnus A
409	20	12	17.	+23	26.	102	<2	
430	21	16	57.	+60	35.	100	6x2	DWD
461	23	21	12.	+58	32.1	13000		Std Source Cassiopeia A

4 Calibration Sources

4.1 Northern Sky

The super strong: Cyg A, Cas A, Tau A, and Vir A.

The 3C Strong Source list: $S_{178} > 100$ Jy.

Cas A is young supernova whose total radio flux is decreasing owing to expansion of the nebula. Vinyajkin (1997) provides a comprehensive study of the low-frequency measurements: 38, 151.5, 290 & 927 MHz. At 151.5 MHz he finds a fractional decrease rate of -0.86 ± 0.09 % per year. Most measurements are done relative to Cyg A. His last value for the Cas A to Cyg A flux density ratio at 151.5 MHz is in 1994.8: 0.94 ± 0.02. Monitoring this ratio with PAPER could provide an update to this value, and potentially even have the precision to see the ratio decrease over a year. Vinyajkin makes the claim that the decrease is not uniform. Small deviations over a decade of $\pm 0.03\%$ are seen, and he explains these as reasonably the result of uneven expansion of some 300 cells, or knots, that constitute some 30% of the nebular emission. Hence, accurate short-term measurements such that we might do with PAPER may show significant different rate than that of the multi-decade average presented in this paper. Measurement with PAPER can easily reach a small fraction of 1% precision. The question will be systematics over time. For example, high precision needs to consider ionospheric absorption effects, which will of course be calibrated to some extent by variations with elevation. Also, the absolute flux ratio of Cas A to Cyg A requires removing the relative beam gain. Accuracy of that will be tested in part by comparison between antenna pairs. Note also that the flux density decay at other frequencies is at different rates as the dynamics of the particles and field lead to variable energy loss rates depending on particle energy (see, e.g., submm study by xx).

4.2 Southern Sky

The southern sky is notable for not having strong sources like Cygnus A and Cassiopeia A. Centaurus A is 10 times weaker than Cygnus A and also at least 10 times larger. It larger than the Cassiopeia A. The Culgoora Array operated at 80 MHz and 160 MHz and its catalog is an important source of flux densities for PAPER. Table **??** provides a list of sources stronger than 100 Jy from the Culgoora Catalog ref Slee et al. c. 1996.

The Molonglo Cross cataloged the southern sky down to very low flux densities at 408 MHz. Burgess & Hunstead have published the Molonglo Southern 4-Jy Sample (MS4; 2006 AJ 131 100) where sources have declinations < -30d. Table **??** extracts from MS4 those sources above 30 Jy, some of which might reach 100 Jy at 150 MHz. The overlap between tables is limited because most strong Culgoora sources are not below -30d declination! Burgess & Hunstead also speak of a "Strong MS4" that would correspond to the northern 3C catalog. These have estimated flux densities at 178 MHz exceeding 10 Jy. As far as I have found in publication and

Table 2: Culgoora Catalog (160 MHz) Sources Above 100 Jy

Source	hh	mm	ss	dd	mm	ss	l	b	Jy	alpha	Other Names
B0433+295	04	33	55.4	+29	35	15	170.6	-11.6	247.0	-0.73	3C 123
B0518-458*	05	18	20.2	-45	49	31	251.6	-34.6	452.0		Pictor A
B0915-118	09	15	42.4	-11	53	13	242.9	25.1	243.0	-1.12	Hydra A; 3C 218?
B1226+023	12	26	32.1	+02	19	14	289.9	64.4	102.0	-0.61	3C 273?
B1228+126	12	28	17.6	+12	39	47	283.8	74.5	566.0	-1.54	Virgo A
B1322-427*	13	22	34.3	-42	44	15	309.5	19.4	1104.0	-0.52	Centauras A
B1648+050	16	48	39.3	+05	04	17	23.0	28.9	378.0	-1.15	Hercules A
B1717-009	17	17	54.1	-00	55	40	21.2	19.6	276.0	-0.49	3C 353?

Table 3: Molonglo Catalog (408 MHz; $\delta < -30$d) Sources Above 30 Jy

Source	hh	mm	dd	mm	Jy	Other Names
B0320-373	03	20	-37	18	259.	
B0407-658	04	07	-65	48	51.1	
B0409-752	04	09	-75	12	36.0	
B0521-365	05	21	-36	30	36.1	
B1322-427	13	22	-42	42	740.	Centaurus A
B1333-337	13	33	-33	42	30.8	
B1814-637	18	14	-63	42	37.0	
B1932-464	19	32	-46	24	39.6	
B2152-699	21	52	-69	54	61.6	
B2356-611	23	56	-61	06	61.2	

in computer archive data bases, these sources are not tabulated. Contact authors. I don't know why Pictor A is not in MS4 (is it galactic?) nor why B0320-373 is not in Culgoora. TBD.

5 Point Source Confusion

In a homogeneous, Euclidean Universe filled with identical sources, the number of discrete ("point") sources stronger than flux density $S_\circ$, $N(S > S_\circ)$ will grow as $S_\circ^{-3/2}$ owing to the increase in volume with R^3 and the reduction in flux density with R^{-2}. Our expanding Universe is filled with a distribution of sources with varying flux densities and spectral indices. The resulting "$\log(N) - \log(S)$" relation depends on $S_\circ$, and, in general, differs from $-3/2$. At meter and centimeter wavelengths the strong, point-source population is dominated by Active Galactic Nuclei (AGN), sources with a wide distribution of specific luminosities (L_ν). Their spectra generally fall with radio frequency as ν^α. The spectral indices α range from -0.4 to -1.0 below 1 GHz, and often flatten above 1 GHz as the emission source changes from "lobes" to "jets," which emanate from central black holes and feed the lobes. The weak-source population (e.g., below 1 mJy at 1 GHz), is dominated by the disk emission of "normal" galaxies, the specific luminosity of which is correlated with the submm/far-IR dust emission. The AGN sources have a wide range of luminosities, and, therefore, are seen across a wide range of redshifts at a given flux-density limit. The fainter galactic emission sources may have a more narrow range of luminosities.

A telescope with limited resolving power will not be able to discern all the point sources on the sky. The measurement of flux density of a source requires some form of "ON" minus "OFF" comparison. Imagine the power detector output of a telescope staring overhead and watching the sky drift by. This is, in fact, what the individual dipoles of PAPER are doing. They detect the galactic synchrotron emission, but little else. There are only a handful of independent samples of the sky during a 24h sidereal day. We can detect "galaxy ON" when the plane is maximally filling the dipole beam pattern (including weighting toward galactic center where the sky is brighter) minus "galaxy OFF" when the high-latitude sky is maximally filling the beam. See §**??**.

As we increase the telescope resolution, more sources become resolved, which is a tautology. The point is that they become resolved with respect to the background that includes all the unresolved sources. Here, we assume that sufficient integration time and bandwidth have been accumulated to reduce system temperature fluctuations to a negligible level (Eq. 10). In the case of PAPER there are two backgrounds: (a) the relatively smooth, continuous distribution of synchrotron emission from our galaxy; and (b) the extragalactic (mostly) point sources. At some resolution the lumps and bumps in (a) will become smaller than the lumps and bumps in (b). I don't know this exactly, but I think the transition is at sub-degree scales at high-latitudes and at radio frequencies around 150 MHz. This is an important exercise in understanding our instrument and its capability. The Haslam synchrotron intensity image of sky is an important starting point that carries information down to about 1 degree scale with accuracy. The fluctuations in sky brightness temperature could be extrapolated to somewhat smaller scales with decreasing accuracy. That is, take a steradian patch of high-latitude sky that we care most about, form a power spectrum or spherical harmonic transform or correlation function, fit power-law index to 1-10 deg range, and extrapolate. [dig up some references on this that appear in the context of CMB power spectrum.] The Kelvin per resolution element can be converted to flux density to compare to point-source confusion estimate that we turn to next.

Perley & Erickson (1984) wrote a scientific justification for 74-MHz receivers on the VLA. In their memo they carefully treated the confusion issue in a series of appendices. I will summarize the salient features of this material, but recommend reading the material more thoroughly. They quote the normalized, differential number counts of Pearson (1975) that are based on two 408-MHz surveys: Cambridge 5C and Molonglo MC1. As discussed above the differential counts depend on flux density:

- Region I: $n(S) = 7600F^{2.2\alpha}S^{-3.2}$, $\quad 10F^{\alpha} < S < 50F^{\alpha}$
- Region II: $n(S) = 1500F^{1.5\alpha}S^{-2.5}$, $\quad 0.8F^{\alpha} < S < 10F^{\alpha}$
- Region III: $n(S) = 1750F^{0.8\alpha}S^{-1.8}$, $\quad 0.001F^{\alpha} < S < 0.8F^{\alpha}$

where S is the flux density in Jy, α is the (negative) spectral index[1], $F = 408/\nu$) is the radio frequency ratio with ν in MHz, and $n(S)$ is the count in sources/steradian. The upper flux density limit is set at 50 Jy, while there are 16 northern hemisphere sources above this; the upper limit is important in confusion calculations below. The 5C counts are valid only down to 0.01 Jy at 408 MHz. The lower limit of 0.001 Jy is used because, quoting Perley & Erickson, "recent number count experiments done at 1400 MHz with the VLA indicate that the extrapolation is valid."

Zeroeth moment (integral number count/sr):

- I $< N >= 3450F^{2.2\alpha}S^{-2.2} - 0.63$
- II $< N >= 1010F^{1.5\alpha}S^{-1.5} - 10$
- III $< N >= 2200F^{0.8\alpha}S^{-0.8} - 1230$

Need to check, but this is probably integral with upper flux density limit evaluated. In the text they evaluate at the lower end giving the total in each range as: 22, 1400, 5.53×10^{5} per steradian, respectively, for their chosen frequency 74 MHz.

First moment (total flux density/sr):

- I $< S >= 6330F^{2.2\alpha}S^{-1.2} - 58F^{\alpha}$
- II $< S >= 3030F^{1.5\alpha}S^{-0.5} - 620F^{\alpha}$
- III $< S >= 11,220F^{\alpha} - 8830F0.8\alpha S^{0.2}$

At the range boundaries, there are $340F^{\alpha}$ Jy/sr above $10F^{\alpha}$ Jy, $2780F^{\alpha}$ Jy/sr above $0.8F^{\alpha}$ Jy, and $9000F^{\alpha}$ Jy/sr down to the bottom of the established counts. Although 0.004% of all sources lie in Region I, they contribute to 3.7% of the total flux density. Similarly Region II has 0.25% of sources and 31% of the flux density. At 74 MHz there are 9.8 Jy per square degree.

[1] Perley & Erickson use the form $\nu^{-\alpha}$ for the scaling of spectra, which is a common, but not favored, definition. The logarithmic spectral index is most clearly defined as the ratio of log of ratios, $alpha \equiv \log(y_2/y_1)/log(x_2/x_1)$. For example, the flux density $y \propto x^{\alpha}$ of synchrotron emission as a function of frequency x, the spectral index is ~-0.5.

Second moment (mean square flux density).

Confusion noise

$$\sigma_c^2 = 0.5\frac{\int S^2 n(S) dS}{\int G^2 d\Omega},$$

where A is the antenna power relative gain pattern.

6 Source Subtraction

6.1 First Effort

<uvcal>. This module was modified to subtract model sources. The standard approach was to start with a data set that had been phase-rotated to a given a priori source position and bandpass calibrated using a short piece of data. This would leave a nominally zero phase response of the source drifting through the primary beam. The source flux density in calibrated Janskies, or uncalibrated counts, was then removed using a version of Peek's analytic beam-pattern software. Small phase gradients would defeat this "real" model. A later modification allowed a catalog of sources to be removed, and the last modification provided an option to write the model into the output data set in place of the difference between input data and model. This routine was abandoned owing to the need to subtract a model to obtain clear "single-source" data for antenna position and delay fitting, which, in turn, would allow accurate modeling of phase needed for accurate subtraction. The module $< uvxecat >$ broke this "can't get started (on subtraction) because can't get started (on calibration)."

6.2 Sun Removal

<*uvexsun*>. This program is no longer being developed. The function of removing the sun from data has been incorporated into <uvexcat>.

6.3 Catalog Subtraction

<*uvexcat*>. This program is used early in data processing for "blind removal" of a source without knowing accurate phase or amplitude calibration. While this is designed to remove a catalog of point sources, it can also squelch the response of the resolved sun. The method is simple: start with zenith-pointing raw data where RA=LST and DEC=LATITUDE; rotate phases to source n and average for selected interval (*interval=*); return to the raw data and remove the (complex) average times a phasor that is the instantaneous phasing used in the averaging step. That is, remove an estimate of the complex amplitude from the data for each source in a catalog. This is simple process, but not accurate because you can't isolate the response of source A with respect to all others on single baseline in a short interval of time; more on this later.

Experience with <uvexcat> to date suggests sequential operation. First, the sun is removed by running with empty catalog; sun removal is default. The default elevation limit is 10d, but this can be changed on the command line (*el_limit=*). Then, I have run through subtracting Cygnus A or Cassiopeia A depending on whether I want to create a data set with the alternate source. If neither, then it is probably best to subtract Cygnus A, the strongest response source after the sun, and then another run subtracting Cassiopeia A. In this third run, other sources such as Taurus A can be included as they and Cas A don't interfere that strongly owing to right ascension separation.

Fig. x shows amplitudes before and after sun removal in the 2006 July data. One sees in the "after" the rise of Cyg A into the primary beam pattern, but then strong beating with Cas A starting around 20h LST, Cyg A transit. Fig. x shows removal of Cyg A that leaves mostly Cas A rising into the beam and exiting. When phases are displayed, these show evidence for Cas A over 20h of sidereal time; Cas A is circumpolar. One also sees in the sun and Cyg A removed Cas A amplitude data various nulls. These are points in time when the fringe rates of the two sources cancel and lead to the Cyg A average "absorbing" the Cas A response—a sidelobe problem resulting from this single baseline effective imaging.

7 Phase Calibration

7.1 Single Source—bx, by

7.1.1 MIRIAD sequence:

- <*c2m*> Convert raw data to MIRIAD format with RFI excision, bias removal.
- <*uvcal*> Create selection of data such as rising part of Cygnus with no autocorrelations and with phasing to Cygnus RA,DEC; needs current version of <*uvcal*>
- <*selfcal*> Find antenna-based amplitudes and phases at "good" frequency window; e.g. 145-150 in 2006 PGB data.
- <*puthd*> Put correct frequency for chosen band in header.
- <*bee*> Find bx, by components with "2pt" option, and save to *antpos* file; needs new version of <*bee*>
- <*uvedit*> Edit *antpos* file into original MIRIAD data base.
- <*uvcal*> Recreate Cygnus data to assure accuracy of fit.

7.1.2 2007 Sep data results

Tabulate Chaitali's analysis vs date and pol, and then DB analysis vs source.

7.2 Single Source—delay

Delay fitting via <*mfcal*> and small program <*delayfit*> This fit includes bz error, cable/receiver delay and, importantly, correlator synch delay. Example from sep data. Cyg–two hours x two pol; no source subtraction

7.3 Single Source—delay,bz

When done for multiple sources, the bz error can be separated from the other, "electrical collimation," terms. That is, the delays can be fit to delay + $bz \sin(\delta)$. Source confusion, RFI and sensitivity have limited this effort to date. The current goal is to proceed with source subtraction and then do the delay, bz fit.

Results from sep07/08. Three sources. delayfit-¿fitbztau.

8 Amplitude Calibration

8.1 Autocorrelation

A priori passband—receiver, cable, cable compensation. Chaitali input. Compare to <mfcal> results on Cyg A with flux density parameters: flux density, reference frequency (GHz) and spectral index. Note that spectral index is defined correctly: $(\nu/\nu_{\rm ref})^{\alpha}$; e.g., Cyg A is something like -0.7 with correct numbers in Baars et al. reference.

Quote cable spec: 10 dB/500 ft at 100 MHz; xx at 200 MHz. MJSales.com.

Plot a priori passband.

Sky spectrum.

$$T_{\rm sys} = T_{\rm r} + T_{\rm syn}(\frac{\nu}{150MHz})^{-2.5}, \tag{14}$$

where $T_{\rm r}$ is approximately 100 K, and $T_{\rm syn}$ varies from xxx K to XXX K (get from autocorrelation and/or Haslam extrapolation). Is that so? What is $A\Omega$ vs frequency? If constant, then λ^2 factor is not canceled.

8.2 Crosscorrelation

<mfcal>: use Cyg flux, spectral index, ref frequency. Get bandpass including any residual system phase. Use <gpcopy> to apply all data.

9 Beam Model

Quote and tabulate Katie's results. Used in abandoned <uvcal> above.

10 Master Fit

Give master equation then for point source model, and discuss big fit that refines delay and antenna positions while fitting for source fluxes and spectra and beam parameters for each antenna.